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TOOTH WEAR EFFECTS ON SHARED DYNAMIC LOADS OF SPUR GEAR PAIRS

BY

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Abstract. The paper presents an analysis of the interaction between the tooth wear and the shared dynamic loads of spur gears. A dynamic wear tooth model is included in the analysis. The effects of addendum modification coefficients on the tooth wear amount and the distribution of the dynamic load are studied for various meshing cycles. The gear dynamic developments are conducted for a model of single-stage geared system which accounts the non-linear time varying mesh stiffness and tooth profile error due to the wear of contacting teeth.

Key words: spur gear, tooth wear, addendum modifications, dynamic loads.

1. Introduction

The meshing accuracy under dynamic conditions is a result of the instantaneous contact conditions between active tooth profiles. The study of the wear of gear contact becomes important in order to predict the change of dynamic behaviour with different operating cycles (Atanasiu & Iacob, 2010; Kuang & Lin, 2001). Most of the approaches on gear wear phenomenon are based on Archald’s wear equation and the wear model developed by (Flodin & Anderson, 1997). On this basis, there were investigated the interaction between

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tooth wear and dynamic loads of spur and helical gears (Yuksel & Kahraman, 2004; Imrek & Duzcukoglu, 2007).

In the present paper, the shared dynamic loads of gear systems is directly related to the deviation of the gear tooth profiles in the form of surface wear and the time-varying mesh stiffness. In the analysis, instantaneous dynamic contact loads are used in wear depth calculations. The effects of the addendum modification coefficient on geometric, cinematic and dynamic characteristics of gear pairs are included in these investigations.

2. Dynamic Model of a Gear Pair

The dynamic model for a gear pair in mesh is shown in Fig. 1. The gear mesh interface is represented by the time-varying mesh stiffness $k_i(t)$ and the viscous damper c. The profile deviations due to the wear process should be included in the composite tooth profile error $e_i(t)$. The differential equations of motion can be expressed as:

$$J_1\ddot{\theta}_1 + c\left(\dot{\theta}_1 r_{b1} - \dot{\theta}_2 r_{b2}\right) r_{b1} + k_i(t)\left(\theta_1 r_{b1} - \theta_2 r_{b2} + e_i(t)\right) r_{b1} = T_1 . \qquad (1)$$

$$J_2\ddot{\theta}_2 - c\left(\dot{\theta}_1 r_{b1} - \dot{\theta}_2 r_{b2}\right) r_{b2} - k_i(t)\left(\theta_1 r_{b1} - \theta_2 r_{b2} + e_i(t)\right) r_{b2} = -T_2 . \qquad (2)$$

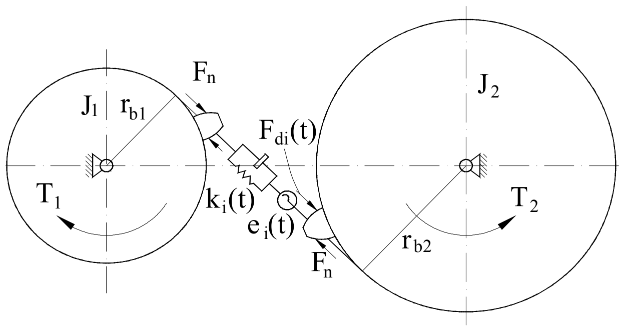


Fig. 1 − Dynamic model of a spur gear pair.

The dynamic normal load between two meshing gear teeth is expressed as

$$F_{di}^{I} = k_i(t)(r_{b1}\theta_1 - r_{b2}\theta_2 + e_i(t)) + c(r_{b1}\dot{\theta}_1 - r_{b2}\dot{\theta}_2) \qquad (3)$$

For a pair of contacting teeth, the time-varying mesh stiffness $k_i(t)$ and the composite tooth profile error $e_i(t)$ are acting as parameter excitations associated with tooth meshing conditions.

The time-varying mesh stiffness is mainly caused by the following factors: (i) the variation of the single mesh stiffness along the line of action; (ii) the fluctuation of the total number of total pairs in contact during the engagement cycle. The effect of bending, shear and Hertzian contact deformation is taking into account in the analytical method to calculate the tooth deformation (Atanasiu, 1998). In the paper, the tooth profile error $e_i(t)$ represents the geometric deviation of a generic gear tooth from the involute profile due to accumulated running wear.

3. Dynamic Wear Model for a Spur Gear Pair

Based on the generalized Archard's model, the wear prediction model proposed by (Flodin, 2000) permits to express the accumulated wear at a contact point on the line of action as follows:

$$h_{1,n} = h_{1,n-1} + \lambda_w \cdot p_H \cdot 2b_H \left(1 - \frac{u_1}{u_2}\right) \cdot n_1 \tag{4}$$

$$h_{2,n} = h_{2,n-1} + \lambda_w \cdot p_H \cdot 2b_H \left(\frac{u_1}{u_2} - 1\right) \cdot n_2 \tag{5}$$

where n_1 and n_2 are the number of the operating cycles for the gears 1 and 2 and $n_2 = n_1 /(\omega_1 / \omega_2)$.

The total wear at a specific point on the line of action results as:

$$h_{s,n} = h_{1,n} + h_{2,n} \tag{6}$$

In order to obtain reliable data for the prediction of the evolution of the tooth wear depths versus the number of running cycles, the parameters which are lumped in eqs. (4) and (5) have to be evaluated using accurately models. Thus, the influences of lubricant, material and surface related parameters are included in the wear parameter λ_w. The wear coefficient λ_w must be taken into account in relation to lubricant film thickness under dynamic conditions. The amount of the wear coefficient λ_w is suggested in relation to the specific film thickness λ_{wo} (Flodin, 2000). In this analysis, a contact wear coefficient $\lambda_{wo} = 2.5 \cdot 10^{-18}$ m^2/N is used (Priest & Taylor, 2000).

The prediction of the minimum film thickness in cylindrical gears is based on the analytical models of the elastohydrodynamic lubrication of line contacts (Hamrock & Jacobson, 1983; Wang & Chen, 1981). The values of lubricant coefficients are the following: the lubricant viscosity $\eta_o = 0.049$ N s/m^2 and the pressure–viscosity coefficient of the lubricant $\alpha = 2.554 \cdot 10^{-8}$ m^2/N.

The wear depth of the tooth profile is calculated according with the updated dynamic load at the running cycles.

4. Results and Discussions

In this analysis, two different gear pairs are considered in order to examine the effects of the addendum modification coefficient on the wear of spur gears. Specifications of the geometrical parameters of the analysed gear pairs are shown in Table 1, where x_1, x_2 are the addendum modification coefficients and the segments AB, AC, AD, AE are the geometrical characteristics used in the dynamic analysis. These parameters are used for spur gear pairs having: number of pinion teeth $z_1 = 25$, number of gear teeth $z_2 = 47$, tooth module $m = 2.5$ mm, face-width of gears, b = 20 mm, and center distance, $a = 90$ mm. The nominal specific load $F_n / b = 200$ N/mm and the pinion speed $\omega_1 = 280\ \mathrm{s}^{-1}$ are considered in the dynamic analysis.

Table 1
Specifications of the Gear Pairs

Gear Pairs	x_1	x_2	AB mm	AC mm	AD mm	AE mm
GP1	−0.3	0.3	5.11	8.12	7.38	12.49
GP2	0.3	−0.3	4.69	4.65	7.38	12.07

The total surface wear $h_{s,n}$ calculate with eq. (6) is used as tooth profile error in eqs. (1), (2), and (3) for dynamic analysis of spur gears.

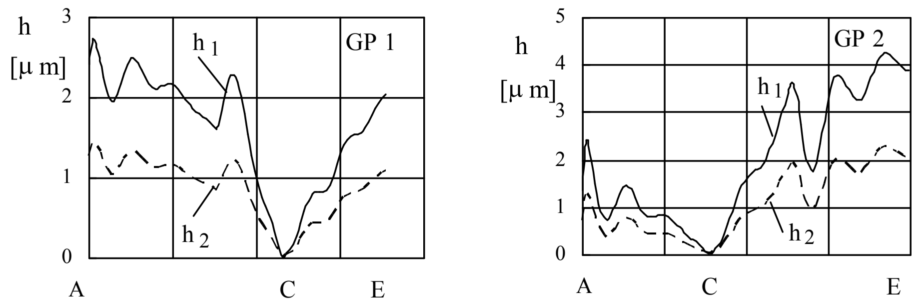
A computer program was developed for simulating the dynamic characteristics of spur gear pairs. The equations of motion are solved by the fourth-order Runge-Kutta method.

The contact pressure, Hertzian band and sliding velocity along the line of action are modified for different values of the addendum modification coefficients. There are being investigated the interaction between these parameters and tooth surface wear, with effect on the shared dynamic load for various meshing cycles.

The variations of the accumulated wear depths along the line of contact are presented in Figs. 2 and 3 in relation with addendum modification coefficients. It can be seen the dynamic effect of the instantaneous contact load on the variation of the wear depth on the engagement cycle. The wear depth is zero at the pitch point, while at the extreme ends of the line of action the amount of the wear is higher. These values are influenced by the distribution of the addendum modification coefficients.

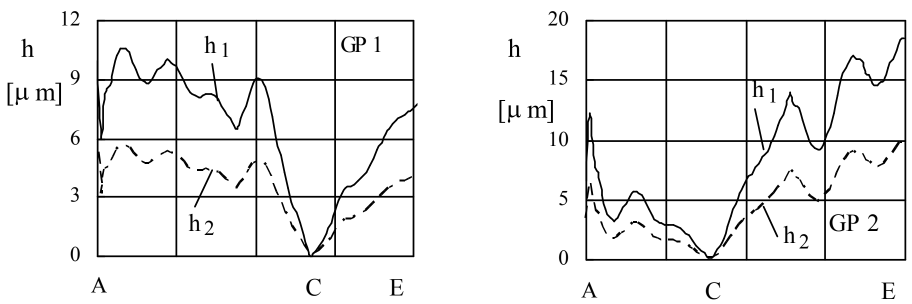
From the distribution curves presented in Figs. 2 and 3 results that, for the same number of the operating cycles, the wear depths of the tooth profiles become higher for the gear pair GP2. These differences of the wear depths point out the impact of the addendum modification coefficients on the distribution of tooth wear depths.

The accumulation of wear causes the change of the distribution of the dynamic contact load and, therefore, the variation of the contact pressure must be updated, according to the number of running cycles.



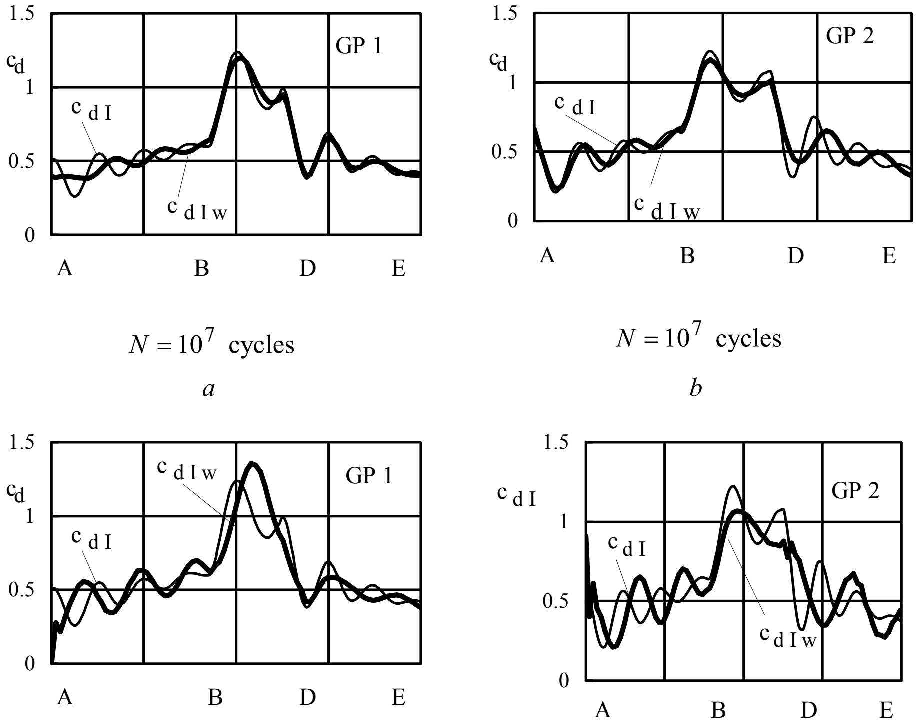
a *b*

Fig. 2 − Variation of the wear depth distributions after $N = 10^7$ operating cycles.



a *b*

Fig. 3 − Variation of the wear depth distributions after $N = 5 \cdot 10^7$ operating cycles.



$N = 10^7$ cycles

a

$N = 10^7$ cycles

b

$N = 5 \cdot 10^7$ cycles

c

$N = 5 \cdot 10^7$ cycles

d

Fig. 4 − Variation of the dynamic load factors after different operating cycles.

The effects of the tooth profile wear on the variation of the dynamic load factor c_{dIw} are shown in Fig. 4 (*a*-*d*) for different number of operating cycles. In these figures $c_{dI} = F_d^I / F_n$, where F_d^I represents the dynamic load corresponding to a single tooth pairs, and $c_{dIw} = F_{dw}^I / F_n$, where the dynamic load F_{dw}^I incorporates the effect of the wear depth of the involute tooth profile.

Numerical results indicate that the amplitude fluctuation of the dynamic factor c_{dIw} is depended of the number of running cycles. For a period until $N = 10^7$ cycles, the factor c_{dIw} presents a slowly increase of the amplitude fluctuation. For $5 \cdot 10^7$ cycles, this fluctuation becomes significant especially for the double tooth contact segments of the meshing path. The addendum modification coefficients have a relevant effect on the amount and variation of the dynamic factor c_{dIw}.

The position of the pitch point C related to the segment BD of the single tooth contact is influenced by the amount of the addendum modification coefficients. Therefore, the variation of the load dynamic factor c_{dlw} corresponding to the single tooth contact is depended by the distribution of the of tooth profile wear and shared normal load on the path of action.

5. Conclusions

The interaction between the tooth wear and the shared dynamic loads of spur gears is presented by using an analytical procedure and numerical simulations. A dynamic wear tooth model is included in the analysis. The following observations should be noted:

1) Tooth wear depth influences the dynamic loads due to increasing the tooth profile error and depends on the number of running cycles. The numerical analysis indicate that the dynamic load factor may be reduced, due to sliding wear in the first period of the investigation examples, until 10^7 operating cycles.

2) The effects of addendum modification coefficients on the variation of the wear depth of tooth profiles and dynamic load of spur gear pairs along the contact line were examined. Evolution of the dynamic load versus addendum modification coefficients is dependant of the number of running cycles.

REFERENCES

Atanasiu V., *An Analytical Investigation of the Time-Varying Mesh Stiffness of Helical Gears*. Bul. Inst. Polit. Iaşi, **XLIV**, *1-2*, s. V, 7−17 (1998).

Atanasiu V., Iacob M.R., *Tooth Wear Effects on Dynamic Transmission Errors of Spur Gears with Addendum Modifications.* International Review of Mechanical Engineering, **4**, *6*, 638−644 (2010).

Bajpai P., Kahraman A., Anderson N.E., *A Surface Wear Model for Parallel Axis Gear Pairs.* Journal of Tribology, **126**, *3*, 597−604 (2004).

Flodin A., Anderson S., *Simulation of Mild Wear in Spur Gears.* Wear, 207, 16−23, 1997.

Flodin A., *Wear of Spur and Helical Gears*. Doctoral Thesis, Royal Institute of Technology, S-100 44, Stochkolm (2000).

Hamrock B.I., Jacobson Bo., *Elastohydrodynamic Lubrication of Line Contacts.* ASLE Transactions, Vol. **27**, *4*, 275–287 (1983).

Imrek H., Duzcukoglu H., *Relation between Wear and Tooth Width Modification in Spur Gears.* Wear, 262, 390−394 (2007).

Kuang J.H., Lin A.D., *The Effect of Tooth Wear on the Vibration Spectrum of a Spur Gear Pair.* Journal of Vibrations and Acoustics, **123**, 311−317 (2001).

Priest M., Taylor C.M., *Automobile Engine Tribology – Approaching the Surface.* Wear, **241**, *2*, 193−203 (2000).

Wang K.L., Cheng H.S., *A Numerical Solution to the Dynamic Load, Film Thickness and Surface Temperatures in Spur Gears.* Journal of Mechanical Design, **103**, 177–194 (1981).

Wojnarowski J., Onishhchenko V., *Tooth Wear Effects on Spur Gear Dynamics.* Mechanism and Machine Theory, **38**, 161−178 (2008).

Yuksel C., Kahraman *A.*, *Dynamic Tooth Loads of Planetary Gear Sets having Tooth Profile Wears.* Mechanism and Machine Theory, **39**, 695−715 (2004).

INFLUENȚA UZĂRII DE FLANC A DANTURII ASUPRA DISTRIBUȚIEI FORȚEI DINAMICE LA ANGRENAJELE CILINDRICE

(Rezumat)

Lucrarea prezintă o analiză a interdependenței dintre uzarea flancurilor dinților în contact şi distribuția forței dinamice la angrenaje cilindrice. Investigațiile realizate au considerat o metodologie specifică în care este inclus modelul dinamic al uzării danturii. Analiza rezultatelor numerice a permis evidețierea influenței numărului de cicluri de funcționare şi a deplasărilor de profil ale danturii asupra mărimii şi variației uzării de flac a danturii şi a coeficientului dinamic.